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AN APPROXIMATE METHOD OF EVALUATING THE TEMPERATURE FIELD OF A SAMPLE IN THERMAL VACUUM MEASUREMENT OF THE MOISTURE CONTENT OF LOOSE MATERIALS

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A model and a procedure are suggested for solving the problem of nonstationary heat conduction for the sample of a moist material in the thermal vacuum method of measuring the moisture content of disperse materials.

In determination of the moisture content of powdered materials by the thermal vacuum method, use is generally made of the "thin-layer" model, where the temperature distribution over the thickness of the material layer is assumed to be uniform [1, 2]. On the one hand, this leads to a simple solution for the sample temperature using the equation of energy balance, and on the other, this assumption introduces a certain error into the result of the temperature measurement, since the temperature drop over the thickness of the material layer reaches several degrees [3]. The studies conducted also confirm the influence of the thickness of the bulk layer on the measurement result [4, 5]. Thus, the necessity arises for a more in-depth study of the temperature field in the sample of the studied material and its influence on the result of measuring the moisture content.

The mathematical model of the process of moisture desorption in the thermal vacuum method of measuring the moisture content is described by a system of partial differential equations for the temperature, pressure, and potential of moisture transfer [6]. In the general case, the differential equations are nonlinear, since the coefficients of transfer are functions of these quantities.

The solution of the problem for the one-dimensional case assuming that the moisture evaporates uniformly over the entire volume of the material and the coefficients of heat and mass transfer do not change in moisture desorption is given in [4, 7]. Here, only the second period of the process of drying was considered, which is justified only at low values of the moisture content. E. S. Krichevskii et al. [8] refined the mathematical model and obtained the solution with allowance for the effect of the side boundaries of the vessel and for the case of high values of the moisture content, when the first period of drying must also be taken into account. The solutions presented in these works, obtained by the methods of integral transforms, are ineffective from the viewpoint of practical use, especially at small times, since they require calculation of a large number of terms of a complicated series. The determination of the temperature at small times is urgent in a rapid dynamic thermal vacuum method of measuring the moisture content [9, 10] that is distinguished by high speed and high metrological characteristics compared to the traditional thermal vacuum method.

Thus, it becomes necessary to develop an engineering technique for calculating the thermal fields in a sample that, on the one hand, would allow one to determine rather simply the temperature field in the sample of the material studied and, on the other, would satisfy the necessary requirements on the accuracy of determining the temperature.

The problem formulated can be solved by an approximate method that is based on combined use of the Laplace integral transform and variational methods [3, 11]. The essence of this approach resides in the fact that first the boundary-value problem is subjected to a Laplace integral transformation and is reduced, with respect

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to the transform, to the solution of a boundary-value problem in the remaining space coordinates. An approximate solution of the boundary-value problem is found by the Bubnov–Galerkin orthogonal method and then an approximate solution of the initial problem is found by passing to the inreverse transform. A numerical analysis of the results obtained shows that the calculation of the temperature for a Fourier number Fo > 0.1 gives good coincidence with the exact value even in the first approximation. This corresponds to the traditional thermal vacuum method when the measurement time is 30 sec or more. The range of the Fourier numbers for the dynamic thermal vacuum method is 0.05 < Fo < 0.1, which makes it possible to obtain the solution for calculation of the temperature to sufficient accuracy only in the third approximation [11]. Correspondingly, it is expedient to use this approach only in the case where the temperature field of the sample is determined in finding the moisture content by the traditional thermal vacuum method, when the extremum of the temperature variation of the sample in its evacuation serves as the informative parameter associated with the moisture content.

The solutions obtained by the integral method of a thermal layer give rather good approximations at both small and large times [12, 13]. These solutions have a simple structure at large values of the Biot number Bi, which again corresponds to the traditional thermal vacuum method, where the coefficient of heat transfer α_T from the sample to the medium is large, which cannot be said about the dynamic thermal vacuum method, because in it the sample is structurally separated from the vapor-air medium by a heat-insulating layer [14].

To solve the formulated thermal problem, the author has selected, as a basis, the method of averaging developed in the 1970s for solving stationary problems in bodies with a complex configuration of the boundaries [15]. This method does not require selection of a system of coordinate functions and works well even in the first approximation [13].

In the present work the author develops further the method of averaging as applied now to nonstationary problems of heat conduction. Let us consider the idea of this method on the example of solving the heat conduction equation for a sample of thickness L whose lower surface is in contact with a temperature sensor and is thermally insulated from the medium:

$$\frac{\partial^2 \vartheta}{\partial x^2} + \frac{W(t)}{\lambda} = \frac{1}{f}\frac{\partial \vartheta}{\partial t}, \quad \left.\frac{\partial \vartheta}{\partial x}\right|_{x=0} = 0, \quad \left.\left(\frac{\partial \vartheta}{\partial x} + h\vartheta\right)\right|_{x=L} = 0,$$

$$\vartheta(t=0) = 0, \quad \vartheta = T - T_{\mathrm{m}}, \quad h = \frac{\alpha_T}{\lambda}. \tag{1}$$

The power of the internal heat sources in the sample $W(t)$ is determined by the amount of moisture evaporated in evacuation of the sample and can be calculated by the formula [2]

$$W(t) = -W_0 \exp(-\alpha_U t), \quad W(t) = U_0 br\gamma\alpha_U. \tag{2}$$

We apply to the differential equation an averaging operator that is found as follows:

$$I_x[\vartheta] = \frac{1}{L}\int_0^L \vartheta(x, t)\, dx = \langle\vartheta\rangle. \tag{3}$$

Owing to linearity, the operator I_x can be applied to the equation of heat conduction term by term:

$$I\left[\frac{\partial^2 \vartheta}{\partial x^2}\right] = \frac{1}{L}\int_0^L \frac{\partial^2 \vartheta}{\partial x^2}\, dx = \frac{1}{L}\left(\left.\frac{\partial \vartheta}{\partial x}\right|_{x=L} - \left.\frac{\partial \vartheta}{\partial x}\right|_{x=0}\right).$$

From the boundary conditions we have

$$\left.\frac{\partial \vartheta}{\partial x}\right|_{x=L} = -h\vartheta|_{x=L} = -h\vartheta_L\,, \quad \left.\frac{\partial \vartheta}{\partial x}\right|_{x=0} = 0\,.$$

Then

$$I_x = \left[\frac{\partial^2 \vartheta}{\partial x^2}\right] = -\frac{h\vartheta_L}{L}\,.$$

We introduce the coefficient ψ by the formula

$$\psi = \psi(t) = \frac{\vartheta_L}{\langle\vartheta\rangle}\,. \tag{4}$$

Calculations showed that we can assume approximately ψ = const, and therein lies the first assumption of the method of averaging. This and subsequent assumptions are substantiated in [13, 15] in detail. Physically, ψ is the ratio of the mean superheating at the boundary to the mean superheating in the region and is termed the coefficient of nonuniformity of the temperature field.

It follows from the theory of a regular mode that in a regular stage of the thermal process ψ is independent of time and is determined by the Biot number and the first eigenvalue for the given body [16].

We apply the operator I_x to the remaining terms of Eq. (1):

$$I_x\left[\frac{1}{a}\frac{\partial \vartheta}{\partial t}\right] = \frac{1}{a}\frac{d\langle\vartheta\rangle}{dt}\,, \quad I_x\left[\frac{W(t)}{\lambda}\right] = \frac{W(t)}{\lambda}\,.$$

Having summed the results of application of the operator I_x to the individual terms of the initial equation, we obtain an ordinary differential equation with respect to $\langle\vartheta\rangle$:

$$\frac{d\langle\vartheta\rangle}{dt} + \frac{ah\psi}{L}\langle\vartheta\rangle + \frac{aW_0}{\lambda}\exp(-\alpha_U t) = 0\,, \tag{5}$$

with the initial condition $\langle\vartheta(0)\rangle = 0$.

The solution of this problem is

$$\langle\vartheta\rangle = \frac{U_0\gamma h}{c(1-\xi\psi)}\left(\exp(-\alpha_U t) - \exp(-\alpha_U\xi\psi t)\right)\,, \tag{6}$$

where $\xi = \alpha_T/(\alpha_U c\gamma L)$.

We make the approximate substitution (the second assumption of the method of averaging)

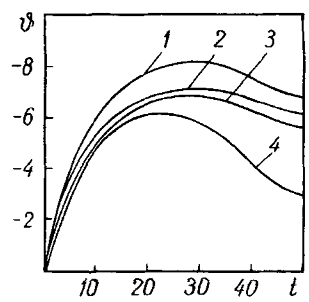
$$\frac{\partial \vartheta}{\partial t} \approx \frac{d\langle\vartheta\rangle}{dt} = -\left(\frac{ah\psi}{L}\langle\vartheta\rangle + \frac{W_0 a}{\lambda}\exp(-\alpha_U t)\right). \tag{7}$$

Having substituted this into (1), we obtain a boundary-value problem for determination of the dependence ϑ_1 in the coordinate x, where ϑ_1 means the first approximation for the temperature of the sample:

$$\frac{d^2\vartheta_1(x,t)}{dx^2} = -\frac{h\psi}{L}\langle\vartheta\rangle\,, \quad \left.\frac{d\vartheta_1}{dx}\right|_{x=0} = 0\,, \quad \left.\left(\frac{\partial \vartheta}{\partial x} + h\vartheta\right)\right|_{x=L} = 0\,. \tag{8}$$

The solution of this problem is

$$\vartheta_1(x,t) = -U\,\varphi_t\,\varphi_x\,, \tag{9}$$



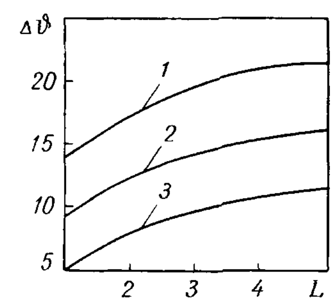


Fig. 1. Change in the temperature of a sample as a function of time: 1, 4) calculation by formula (13) for L = 5 and 2 mm, respectively; 2, 3) calculation by formulas (6) and (14), respectively, for L = 5 mm. ϑ, K; t, sec.

Fig. 2. Dependence of the error of calculation of the temperature for the model of a "thin layer" of a sample on the layer thickness at different instants of time: 1) 50 sec; 2) 30; 3) 10. $\Delta\vartheta$, %; L, mm.

where

$$U = \frac{h\psi}{L}\frac{U_0 rb}{c(1-\xi\psi)}; \quad \varphi_x = \frac{x^2}{2} - L\left(\frac{1}{h} + \frac{L}{2}\right);$$

$$\varphi_t = \exp(-\alpha_U t) - \exp(-\alpha_U \xi\psi t). \tag{10}$$

The coefficient ψ can be defined more accurately as the ratio of the integral-mean superheating at the boundary to the integral-mean superheating in the region [13]:

$$\psi = \frac{\frac{1}{t^*}\int_0^{t^*} \vartheta(x=L, t)\,dt}{\frac{1}{t^*}\int_0^{t^*}\langle\vartheta(t)\rangle\,dt} = \frac{\frac{1}{t^*}\int_0^{t^*} U\varphi_x(x=L)\,\varphi_t\,dt}{\left(\frac{1}{t^*}\int_0^{t^*} U\,dt\right)\left(\frac{1}{L}\int_0^{L}\varphi_x\,dx\right)}. \tag{11}$$

Knowing the dependence $\varphi_x = \varphi_x(x)$ (10) and having performed simple calculations, we write the final expression for ψ:

$$\psi = \frac{1}{1 + \frac{\mathrm{Bi}}{3}}, \tag{12}$$

where $\mathrm{Bi} = \alpha_T L/\lambda$.

Thus, we have obtained the first approximation of the solution of problem (1). As was shown by [13, 15], the method of averaging, even in the first approximation, allows one to obtain a result whose error does not exceed 1.5%.

We write the formula for $\vartheta(x, t)$ at $x = 0$ with account for (12):

$$\vartheta_1(0, t) = \frac{U_0 rb}{c\left(1 - \frac{\xi}{1 + \frac{\mathrm{Bi}}{3}}\right)}\frac{1 + \frac{\mathrm{Bi}}{2}}{1 + \frac{\mathrm{Bi}}{3}}\left[\exp(-\alpha_U t) - \exp\left(-\frac{\alpha_U \xi t}{1 + \frac{\mathrm{Bi}}{3}}\right)\right]. \tag{13}$$

The expression for $\vartheta(t)$ in the case of the "thin-layer" model can be obtained from (13) under the condition Bi = 0:

$$\vartheta(t) = \frac{U_0 rb}{c(1-\xi)} [\exp(-\alpha_U t) - \exp(-\alpha_U \xi t)] . \tag{14}$$

Using formulas (6), (13), and (14), we calculated ϑ for a sample of quartz sand with different thicknesses of the layer L as a function of time. Here, the following initial data were taken: $U_0 b = 0.3\%$, $r = 2.4 \cdot 10^6$ J/kg, $c = 800$ J/(kg·K), $\gamma = 1500$ kg/m^3, $\lambda = 0.33$ W/(m·K), $\alpha_T = 70$ W/(m^2·K), $\alpha_U = 0.1$ sec^{-1}.

Figure 1 presents graphs of the change in the temperature of the sample as a function of time. It is seen from the graphs that use of formula (6) does not give a noticeable gain in accuracy compared to the model for a "thin layer" of the sample (14); the error in determining ϑ by formula (14) increases with the thickness of the bulk layer of the sample L (Fig. 2). The error $\Delta\vartheta$ was found using the expression

$$\Delta\vartheta = \left| \frac{\vartheta - \vartheta_1}{\vartheta_1} \right| \cdot 100\% . \tag{15}$$

We can note that the value of $\Delta\vartheta$ increases with time, and therefore calculation of the sample temperature by the "thin-layer" model can hardly be considered as justified in the case of the traditional thermal vacuum method, since the measurement time here can be several minutes [2]. Therefore, it is recommended that the temperature field in the sample be analyzed using (9), (10), and (12).

NOTATION

U_0 and U, initial and current moisture content of the material; T, current temperature, K; T_m, temperature of medium; λ, thermal conductivity, W/(m·K); a, thermal diffusivity, m^2/sec; ϑ, superheating, K; c, specific heat of the sample, J/(kg·K); r, heat of vaporization, J/kg; γ, density of the sample, kg/m^3; α_U, coefficient of the rate of moisture desorption, sec^{-1}; b, coefficient of completeness of moisture extraction; α_T, coefficient of heat transfer, W/(m^2·K); L, thickness of the sample, m.

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